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# A REVIEW ON CONTROLLER DESIGN OF LOW GAIN WITH POLE PLACEMENT CONSTRAINTS

**Jitendra Prajapati [1] and Chirag Gupta[2]**
Scholar, M.Tech(P.E)., VIT, Bhopal, India [1]
Asst. Professor, Electrical &Elex., VIT, Bhopal, India[2]

## ABSTRACT

This paper addresses the review on controller design of low gain with placement constraints. Design of state- or output feedback *H,* controllers that satisfy additional constraints on the closed-loop pole location. Sufficient conditions for feasibility are derived for a general class of convex regions of the complex plane. These conditions are expressed in terms of linear matrix inequalities (LMI's), and our formulation is therefore numerically tractable via LMI optimization. In the state-feedback case, mixed *H, / H,* synthesis with regional pole placement is also discussed. Finally, the validity and applicability of this approach are illustrated by a benchmark example.

**Key words:** Constraints, LMI

## INTRODUCTION

ANY classical control objectives such as disturbance attenuation, robust stabilization of uncertain systems, or shaping of the open-loop response can be expressed in terms of *H,* performance and tackled by Ha-synthesis techniques [lo], [12]. Since it only involves solving two Riccati equations, *H,* synthesis has a low complexity comparable to that of linear-quadratic-Gaussian (LQG) synthesis [ 111. However, *H,* design deals mostly with frequency-domain aspects and provides little control over the transient behavior and closedloop pole location. In contrast, satisfactory time response and closed-loop damping can often be achieved by forcing the closed-loop poles into a suitable subregion of the left-half plane (see, for instance, the discussion in [I]). In addition, fast controller dynamics can be prevented by prohibiting large closed-loop poles (often desirable for digital implementation).

One way of simultaneously tuning the *H,* performance and transient behavior is therefore to combine the *H,* and poleplacement objectives.

Decentralized control approach has been become the most popular and preferred control strategy for large scale system for over many past years [1,2]. The overview for analysis and solving methods for design problems can be found in [3]. In the decentralized control the whole system is considered as interconnection of subsystems.

A local controller is designed for individual one based on available local information, which ensures stability and performance depending on one requirement| the local objective. The global objective is the designed local controllers must ensure the stability and the performance of the whole system.

The choice of subsystems a_ects the performance of control system, which is limited by information structure constraints [4,5]. Based on information structure constraints, decomposition of large scale system is the fundamental pre-requisite step for control designing for breaking a large dimensional system into smaller subsystems [6].

Control design for a system with overlapping subsystems is started by expanded the system into large dimensional, where the subsystems are appeared as disjoint [5, 6, 8]. The expanded space contains all the necessary information of the original system such that a control law is designed for each subsystem, then contracted back for implementation of control law into original system. For the expansion and contraction of the system, the mathematical framework is known as inclusion principle [4, 6].

A good damping response and fast decay rate can be imposed on the system by restricting the closed loop eigen values under the region of intersection of conic sector and a shifted half plane in the complex s-plane [11]. Such a restriction implies that all closed loop poles lie under D-region. This is also known as D-stability of the system and the technique is known as regional pole placement.

By satisfying regional pole placement constraints, a controller is able to guarantee satisfactory transient performance. The regional pole placement with the other design constraints is considered in [9, 10, 12, 13].

Recently, LMIs have become a powerful tool for solving numerous control problems.

The LMIs are convex optimization problem, which can be solved e_- ciently [14, 15]. Control problems such as Lyapunov stability, Reccati inequality etc. can be easily written as LMIs and also multiple LMIs can be written as single LMI with larger dimension. Thus LMIs help for solving a variety of optimization and control problems. Generally the state feedback control problem is expressed as bilinear matrix inequalities (BMIs) optimization problem [16]. One approach for solving this BMIs problem is to convert it into LMIs problem with addition constraints [16]. Another approach is to solve LMIs problem iteratively. Homotopy approach for solving BMIs optimization problem is one example of second type approach. A path-following method for solving BMIs problem is considered in [17], where BMIs problem is solved by introducing _rst order approximation into the control variables. This results LMIs optimization problem by neglecting bilinear term which is assumed as a small quantity.

Then the resulted LMIs problem is solved for perturbation term that slightly improves the performance of controller. The whole process is repeated until an optimum solution for controller is achieved. In homotopy method [18] for solving decentralized overlapping control problem based on two homotopy path-followings. Along _rst path the centralized controller is deformed into decentralized controller in each step. Along second path the decentralized control design problem is linearized and solved. Conventional methods for load frequency control (LFC) of interconnected power system are PI and PID control, which has wide application in industries. However, their controller parameters are determined by trial and error methods.

**Decentralized Controller**

While dealing with control problems three steps: modeling, describing qualitative properties and controlling system behaviors are applied. This concept is applicable for centralized control, where a single controller is designed based on whole system information. But centralized control is not reliable and economical for the implementation into large scale system and also increases complexity in the design process. Because there is possibility of losing local data, presence of time delays due to long distance information transfer and presence of uncertainty in the model. Thus, the control problem becomes too large to be controlled and too complex to be solved.

Whereas decentralized approach [2, 7] provides a way to deal with above difficulties by breaking the original system into a no. of subsystem. controlled by a local controller, which requires a part of global information. Thus decentralized control design solves difficulties encountered in analyzing, designing and implementing control strategies and algorithms in centralized case.

**LMI FORMULATIOOFN P OLE-PLACEMENT OBJECTIVES** This section discusses a new LMI-based characterization for a wide class of pole clustering regions as well as an extended Lyapunov Theorem for such regions. Prior to this presentation, we briefly recall the main motivations for seeking pole clustering in specific regions of the left-half plane. It is known that the transient response of a linear system is related to the location of its poles [24], [I]. For example, the step response of a second-order system with poles X = -<*w*, f *jwd* is fully characterized in terms of the undamped natural frequency *wn* = (XI, the damping ratio (, and the damped natural frequency *Wd*.

By constraining X to lie in a prescribed region, specific bounds can be put on these quantities to ensure a satisfactory transient response. Regions of interest include a-stability.

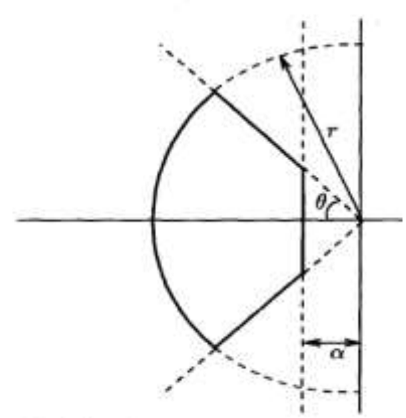


Fig. 1. Region $S(\alpha, r, \theta)$.

dynamical system $x = Ax$ is called %table if all its poles lie in *D* (that is, all eigenvalues of the matrix *A* lie in *D)*. By extension, *A* is then called D-stable. When ***D*** is the entire left-half plane, this notion reduces to asymptotic stability, which is characterized in LMI terms by the Lyapunov theorem.

Specifically, ***A*** is stable if and only if there exists a symmetric matrix *X* satisfying

$$AX + XA^T < 0, \qquad X > 0.$$

This Lyapunov characterization of stability has been extended to a variety of regions by Gutman [19]. The regions
considered there are polynomial regions of the form.

$$\mathcal{D} = \left\{ z \in \boldsymbol{C} : \sum_{0 \le k,l \le m} c_{kl} z^k \overline{z}^l < 0 \right\}$$

Note that (4) is derived from (3) by replacing *zkZl* with *AkX(AT)* 'T. he Lyapunov theorem is clearly a particular case
of this result. Unfortunately, controller synthesis based on Gutman's characterization is hardly tractable due to the polynomial nature of (4). Recall that the closed-loop state-space matrices depend affinely on the controller state-space data [7], [14]. To keep the synthesis problem tractable in the LMI framework, it is necessary to use a characterization of pole clustering that preserves this affine dependence. In other words, it is necessary to use conditions that are affine in the state matrix *A,* such as the Lyapunov stability condition (2). Apart from a few special cases, there is no ystematic way of turning (4) into an LMI in *A.* In fact, polynomial regions are not necessarily convex. With
controller synthesis in mind, this motivates one to look for an alternative LMI-based representation of D-stability regions.

***LMl Regions***

The class of LMI regions defined below turns out to be suitable for LMI-based synthesis. Hereafter, @ denotes the Kronecker product of matrices (see, e.g., [IS]), and the notation *M = [pkl]15k,lsm* means that *M* is an *m* x *m* matrix (respectively, block matrix) with generic entry (respectively, *Definition* 2.1 *(LMZ Regions):* **A** subset ***D*** of the complex plane is called an LMI region if there exist a symmetric matrix Q = *[ a k l ]* E ***R"'"*** and a matrix *p* = ***[&l]*** E *Rmx"* such that

$$\mathcal{D} = \{ z \in \boldsymbol{C} : \quad f_{\mathcal{D}}(z) < 0 \}$$

**with**

$$f_{\mathcal{D}}(z) := \alpha + z\beta + \overline{z}\beta^T = [\alpha_{kl} + \beta_{kl} z + \beta_{lk} \overline{z}]_{1 \le k,l \le m}.$$

Note that the characteristic function *f~* takes values in the space of *m* x *m* Hermitian matrices and that *"<O"* stands for In other words, an LMI region is a subset of the complex plane that is representable by an LMI in *z* and Z, or equivalently, an LMI in *x* = Re(z) and y = Im(z). **As** a result, LMI regions are convex. Moreover, LMI regions are symmetric with respect to the real axis since for any ***z*** E ***D,*** Interestingly, there is a complete counterpart of Gutman's theorem for LMI regions. Specifically, pole location in a given LMI region can be characterized in terms of the *m* x *m* block matrix.

***Theorem 2.1****:* The matrix *A* is D-stable if and only if there exists a symmetric matrix *X* such that
*MD(A, X )* < 0, X > 0. (8)

Note that *MD(A, X )* in (7) and *f D ( z )* in (6) are related by the substitution (X, *A X , X A T )* **t-)** (1, *z, 2).* As an example, the disk of radius *T* and center *(-4,* 0) is an LMI region with characteristic function.

***Extent of the Class of LMI Regions***

Theorem 2.2 answers our need for a characterization of stability regions that is affine in the *A* matrix. The convenience of LMI regions for synthesis purposes will be illustrated in the next two sections. Yet the case for LMI regions would be incomplete if this class of regions was insufficiently large. In the remainder of this section, we show that LMI regions not only include a wide variety of typical clustering regions, but also form a dense subset of the convex regions that are symmetric with respect to the real axis. In other words, LMI regions cover most practical needs for control purposes.

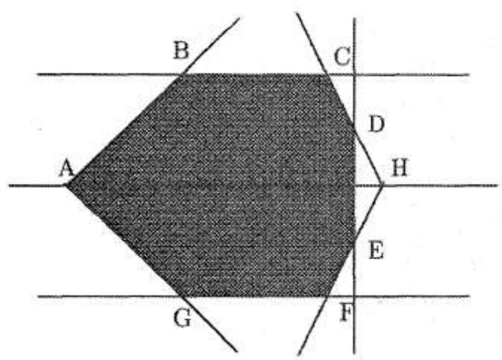


Fig. 2. **Convex** polygon as an intersection of LMI regions.

Let us first review some simple LMI regions. Considering functions of the form shows that conic sectors *ax* + *b* < - 1 ~ 1 , vertical half planes *z*+*a* < 0 or *x*+*a* > 0, vertical strips *hl* < ***IC*** < *hz,* horizontal strips IyI < *w,* as well as ellipses, parabolas, and hyperbolic sectors are all LMI regions.

To derive more complex regions, observe that the class of LMI regions is invariant under set intersection. Specifically, given two LMI regions *DI* and *Vz* and their associated characteristic functions fDl and ***f D 2*** **,** the intersection *D* = D1 n *Dz* is also an LMI region with characteristic function

The BMI problem is linearized by using a _rst order perturbation approximation. Then the resulting LMI problem is solved to compute the perturbation term which slightly improves the performance and has value small enough so that perturbed variables satisfy the initial BMI problem. The centralized solution of control problem is taken as initial solution for starting iterative algorithm.

**Application Problems**

**Load Frequency Control**

In an interconnected power system power demand changes according to end users, this directly a_ects frequency and tie line power ow. The objectives of load frequency control (LFC) are to minimize the deviations in frequency and tie line power ow and to maintain steady state errors zero.

**Formation Control**

In formation control [25], a group of unmanned aerial vehicles move in a specified pattern, where may exist one or more leader and other followers. Different control strategies can be adopted depending on speci_c information structure constraints, to control the whole system.

*Corollary* 2.1: Given two LMI regions D1 and *Dz,* a matrix ***A*** is both D1-stable and D2-stable if and only if there exists a positive definite matrix *X* such that *MD ~ (* AX,) < 0 and *Proof:* Simply observe that *M D ~ ~ D X~ )( A*= , D iag *(MD ~ ( AX ,) ,M D ~ ( AX, ) )* since *D1* n *D2* has the characteristic function Diag *( f D , , f D , )* . Applying Theorem 2.2 to *MD,(A, X )* < 0. D1 n *D2* completes the proof.

## CONCLUSIONS

From the the survey of related literature, we see that unstructured *H2* , *H',* and eigenvalue (and eigenstructure somehow) assignment have been solved. Efficient algorithms for *H2* and *H,* optimization with static output or reduced-order feedback have also been developed. Static output eigenvalue and eigen structure assignment are also clear. However, decentralized problems are generally open. Approaches have been proposed for the decentralized *H2* and work well in practice, but convergency requires some strong conditions or are generally dependent on the initial guess. No efficient method is known for the decentralized *H,* problem.

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